ERRORS OF A STRAPDOWN GYROCOMPASS FOR OBJECTS WITH UNLIMITED TURNING ANGLES*

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Abstract

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Differential equations for a strapdown gyrocompass (SG) based on quaternion parameters brought to the axes of the horizon frame, with horizontal correction from accelerometers and Schuler frequency tuning, have been constructed. The equations of SG errors have been derived and analyzed. The experimental research that was conducted on the SG trial model produced at "Optolink" company both in stationary conditions and onboard a mobile object has confirmed the operability of the developed algorithms.

Introduction

Strapdown gyrocompasses are widely used on land, air, and marine mobile objects. Their cost and the accuracy of output parameters play significant role in their usage. A strapdown gyroscopic compass is realized on the basis of a strapdown inertial navigation system, so the description of the process of SG functioning is substantially based on the theory of inertial attitude control and navigation systems. Gyrocompasses based on a strapped down inertial navigation system have methodical errors as well as instrumental errors which basically depend on the errors of the sensors. Methodical errors are connected with strapdown gyrocompass operational algorithms, which is why there is a need for research of peculiarities of their functioning in order to increase the accuracy of course angle determination. Instrumental errors of sensors also require a careful research because of the significant contribution to the errors of strapdown gyrocompasses. However in the conditions of inability of invoking of the additional external information (an autonomous strapdown gyrocompass) there are significant complexities in the problem of identification of instrumental errors. Nevertheless it is possible to increase an accuracy of a strapdown gyrocompass at the expense of application of algorithmic solutions at various stages of its functioning.

The theme urgency is determined by necessity to estimate a heading angle for the mobile objects with unlimited turning angles. For example, in gyroinclinometers, in bords, shafts, tubes with vertical and horizontal sections, in the mobile robots, etc. Existing gyrocompasses using Euler-Krylov's angles or Euler's kinematic equations do not ensure the solution of this problem. In this report the operation algorithms of a SG in the form of the quaternion kinematic equations [1-3] are applied as a solution of the problem.

The results of the theoretical and experimental research

The strapdown gyrocompass is realized on strapdown inertial navigation system's base, so the description of its functioning process is largely founded on an appropriate theory. Whereas mathematical models and firmware of the gyrocompasses are a commercial property and aren't subjected to publication, there is a necessity of development of new types of its operation algorithms. The element base is composed from three fiber-optic gyros, three compensating type quartz accelerometers, and high-efficiency onboard computer.

The objective of this paper is a derivation of SG error equations based on the quaternion differential equations having a tuning on the Schuler's frequency. The quaternion equations having an important property of a solution nondegenerablity in a counterweight to the classical Euler's kinematic equations are used.

In order to reach the formulated objective the following problems have solved in this work:

- strapdown inertial navigation system's heading channel mathematical model and functioning algorithms including the terms of correction from the accelerometers for a horizon simulation and heading angle determination are developed;
- the algorithm of an initial alignment of the gyrocompass is realized;
- the error equations are derived.

The SG differential equations using the quaternion parameters, with transformation to horizon frame axes, and with horizontal correction from the accelerometers, and with the Schuler's frequency tuning eliminating the ballistic deviations during a horizontal accelerations [4] are applied. Thereby:

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1. Onboard the mobile object the horizon axes are modeled in an on-board computer.

2. Selectivity of one of horizon frame axes to the North direction (on the stationary basis) or to a direction of a vector of an angular velocity of the geographical frame lying in a horizon plane is ensured on the mobile object.

Horizontal correction terms provide a bringing of a horizon frame modeled in an on-board computer to a horizon plane, at that there is a feedback by angles of the modeled frame inclination relatively a horizon plane. During an initial alignment mode and normal functioning the Schuler's conditions are ensured, so there's no accumulation of errors produced by bias of a three-component gyroscopic angular rate measurer and a three-component specific force measurer. At the same time the system is opened by the channel of an azimuth, so bias of a three-component gyroscopic angular rate measurer lead to accumulation of errors in yaw angle estimation. Estimations of current yaw, pitch, roll and heading angles are continuously determined by use of trigonometric formulas on the basis of quaternion components and come to the navigation unit and to control channels of mobile objects.

The course angle calculation is based on the formula $\Psi = \psi + \Psi_K$, where Ψ_K is an angle between the horizon orthodromical accompanying frame Oη and the geographical accompanying frame Oζ. So the course estimation is

$$\hat{\Psi} = \hat{\psi} + \hat{\Psi}_K, \quad tg\hat{\Psi}_K = \frac{-\omega_{\eta 3}^k}{-\omega_{\eta 1}^k},$$

here $\hat{\Psi}_K$ is an angle Ψ_K estimation; $\omega_{\eta 1}^k, \omega_{\eta 3}^k$ are angular rates of the correction transformed to the axes of the horizon frame.

In the case of an angular motion of mobile object, its influence is eliminated by transformation of angular rates and accelerations to a horizon plane with accuracy of errors of a modeled horizon frame. The same is required for fulfillment of the Schuler conditions.

The initial latitude is estimated by signals of an angular rate and specific force sensors during an initial alignment. The initial alignment is made for a mobile object which is stationary with respect to the Earth. Estimated angle of current latitude is used in an azimuthal correction in orientation algorithms during the process of normal functioning.

Gyrocompass functioning develops of three main stages (an initial alignment using trigonometrical formulas, an accelerated alignment using the differential equations and directly the operational mode), so performance of the analysis of errors of an autonomous gyrocompass is required at each stage separately.

The SG errors $\Delta\Psi, \Delta\theta, \Delta\gamma$ (errors of course angle, pitch angle and roll angle accordingly) during an initial alignment stage using trigonometrical algorithms directly depend on instrumental and methodical errors of inertial sensors (gyros and accelerometers) and are derived on the basis of a variation of equations for an initial alignment. Formulas for errors in projection on the geographical frame axes look like

$$\Delta\Psi = \Delta\Psi_K = \frac{\Delta\omega_{\zeta 3}}{U\cos\varphi} + \frac{\Delta W_{\zeta 3}\cos\Psi\, tg\theta}{g} + \frac{\Delta W_{\zeta 1}\sin\Psi\, tg\theta}{g} tg\varphi - \frac{\Delta W_{\zeta 3}}{g} tg\varphi,$$

$$\Delta\theta = \frac{\Delta W_{\zeta 1}\cos\Psi - \Delta W_{\zeta 3}\sin\Psi}{g}, \quad \Delta\gamma = -\frac{\Delta W_{\zeta 3}\cos\Psi + \Delta W_{\zeta 1}\sin\Psi}{g\cos\theta},$$

here $\Delta\omega_{\zeta i}, \Delta W_{\zeta i}\ (i = 1,2,3)$ are fiber-optic gyros and accelerometers biases transformed to the geographical frame; U is angular rate of rotation of the Earth; φ is a local latitude.

At this stage the important role is played by a filtration of signals of sensors as at the expense of it an influence of noise of signals of gyros and accelerometers is considerably reduced. Run to run repeatability of output parameters of a SG is increased as a result.

An error of course angle determination during an operational stage depends on an error of determination of horizontal components of speed $\Delta v_{\zeta 1}$ and $\Delta v_{\zeta 3}$, and error of determination of latitude angle $\Delta\varphi$, and error of determination of local vertical line α_ζ and χ_ζ, and also on vertical component of angular rate of drift of gyros in the geographical frame.

The SG error during an operational mode is determined by the equation:

$$\Delta\Psi_K = \frac{R\left(v_{\zeta 3} + RU\cos\varphi\right)\left(\dot{\chi}_\zeta - \alpha_\zeta U\sin\varphi - \alpha_\zeta v_{\zeta 3} tg\varphi / R + \Delta\omega_{\zeta 3}\right)}{v_{\zeta 1}^2 + v_{\zeta 3}^2 + RU\cos\varphi\left(2v_{\zeta 3} + RU\cos\varphi\right)} +$$

$$\frac{Rv_{\zeta 1}\left(\dot{\alpha}_\zeta + \chi_\zeta U\sin\varphi + \chi_\zeta v_{\zeta 3} tg\varphi / R + \Delta\omega_{\zeta 1}\right)}{v_{\zeta 1}^2 + v_{\zeta 3}^2 + RU\cos\varphi\left(2v_{\zeta 3} + RU\cos\varphi\right)} - \Delta\psi + \left(\alpha_\zeta\cos\Psi - \chi_\zeta\sin\Psi\right)\tan\theta.$$

Part of east component of angular rate of the drift has significant contribution. At the presence of constant velocity the part of northern component of angular rate of the drift has influence. The greater relative velocity horizontal projection values, the greater that influence. In addition, there is an appreciable impact of a leveling error.

Static characteristics of the SG by angles of course $\hat{\Psi}$, pitch $\hat{\theta}$ and roll $\hat{\gamma}$ has been evaluated under laboratory conditions. In order to do this the strapdown gyrocompass has been mounted on the platform of turn table КПА-5 which was aligned in horizon plane with accuracy of 6', and the device centerline was aligned with a north direction with an error not more than 0.1°. Three times the platform turns serially by all three angles from a starting position. Mean value of error of static characteristics by angle $\hat{\Psi}$ is presented on fig. 1, where $\max\Delta\Psi$ is 0.34°.

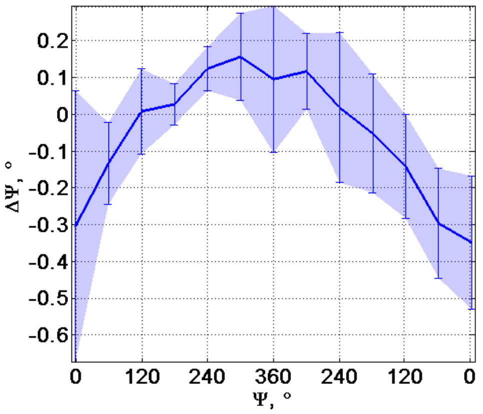

Fig. 1. An error of the static characteristic of the strapdown gyrocompass

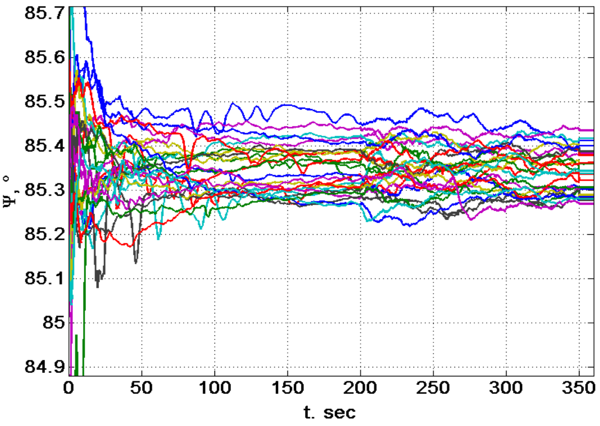

Fig. 2. Run to run repeatability of indications of the strapdown gyrocompass

The estimation of run to run repeatability of indications of the SG has been made. In a series of 26 runs under static conditions the variation of course angle calculation (fig. 2) was 0.3° (σ_Ψ=0.054°). A standard deviation of noise of gyros was $\sigma\omega_x$=0.06÷0.5°/hr ($\sigma\omega_x$=0.01÷0.02°/hr after a filtration by moving average).

Conclusions

1. The mathematical model of the strapdown gyrocompass has been developed due to algorithms of gyrocompassing in the form of the relation of east and northern horizontal correction components included into the quaternion differential equations.
2. The equations of errors of the strapdown gyrocompass have been derived. The analysis has shown that the most contribution to an error of course angle determination makes the east axis component of angular rate of drift, the inexact initial alignment (an angle of an error of modeled geographical frame leveling), and also, at the presence of motion of mobile object, an influence of northern axis component of angular rate of drift increases. In addition, speed error reaches the significant values.
3. The trial model of the strapdown gyrocompass using the developed algorithms of the gyrocompassing has been experimentally researched, and thus the theoretical backgrounds of its functioning have been confirmed.

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